POINTS, LINES, AND PLANES

The terms **point**, **line**, **plane**, **distance along a line**, and **distance around a circular arc** are all left as **undefined terms**; that is, they are only given intuitive descriptions. For example, a point can be described as a location in the plane, and a straight line can be said to extend in two directions forever. It should be emphasized that, while we give these terms pictorial representations (like drawing a dot on the board to represent a point), they are concepts, and they only exist in the sense that other geometric ideas depend on them.

<table>
<thead>
<tr>
<th>Term</th>
<th>Diagram/Symbols</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Point</strong> - A location that has neither shape nor size</td>
<td>![Point Diagram]</td>
</tr>
<tr>
<td><strong>Line</strong> - Made up of points, has no width. There is exactly one line through any two points.</td>
<td>![Line Diagram]</td>
</tr>
<tr>
<td><strong>Plane</strong> - A flat surface made up of points that extend infinitely in all directions. There is exactly one plane through any three points.</td>
<td>![Plane Diagram]</td>
</tr>
<tr>
<td><strong>Collinear</strong> - Points on the same line.</td>
<td>![Collinear Diagram]</td>
</tr>
<tr>
<td><strong>Coplanar</strong> - Points in the same plane.</td>
<td>![Coplanar Diagram]</td>
</tr>
<tr>
<td><strong>Intersection</strong> - The intersection of two or more geometric figures is the set of points they have in common.</td>
<td>![Intersection Diagram]</td>
</tr>
<tr>
<td><strong>Line Segment</strong> - A measurable part of a line consisting of two endpoints and all points in between.</td>
<td>![Line Segment Diagram]</td>
</tr>
<tr>
<td><strong>Ray</strong> - A part of a line with one endpoint and all points extending away from the endpoint.</td>
<td>![Ray Diagram]</td>
</tr>
</tbody>
</table>
**Figure** - A set of points in a plane.  
(Usually the term figure refers to certain common shapes like triangle, square, rectangle, etc. But the definition is broad enough to include any set of points.)

**Geometric Construction** - a set of instructions for drawing points, lines, circles and figures in the plane. Constructions are drawn with a pencil, compass and straight edge.

**Length of a Segment** - The length of the segment \( \overline{AB} \) is the distance from \( A \) to \( B \) and is denoted \( |AB| \) or \( AB \). Thus, \( AB = \text{dist}(A, B) \).

---

**Examples**

*Use the figure to name each of the following.*

1. A line containing point \( W \).
   
   \[
   \vec{VW}, \quad \vec{VY}, \quad \vec{WX}
   \]

2. A plane containing point \( X \).
   
   \[
   \text{plane } P, \quad \text{plane } XYZ, \quad \text{plane } VZW
   \]

3. A line containing point \( T \).
   
   \[
   \text{line } r, \quad \text{line } TX
   \]

*Use the number line to find the following measurements.*

4. \( TW \)
   
   \[
   |-5 + 4| = 9
   \]

5. \( \text{dist}(S, V) \)
   
   \[
   |-7 - 1| = |-8| = 8
   \]

6. \( ST + VW \)
   
   \[
   3 + 4 = 7
   \]
Construct a line segment equal in length to segment $\overline{MN}$:

![Diagram of segment MN]

Construct a segment that is twice the length of segment $\overline{MN}$:

![Diagram of segment twice MN]

**Draw and label a figure for each relationship.**

1. Lines $AB$ and $CD$ intersect at $E$ for $A (-2, 4)$, $B (0, -2)$, $C (-3, 0)$, and $D (3, 3)$ on a coordinate plane. Point $F$ is coplanar with these points, but not collinear with $\overline{AB}$ or $\overline{CD}$.

![Diagram of intersecting lines and points]

2. $\overline{QR}$ intersects plane $T$ at point $S$.

![Diagram of QRS and plane T]

3. Line $\ell$ lies in plane $N$ and contains point $L$.

![Diagram of line l in plane N]
SEGMENTS

<table>
<thead>
<tr>
<th>Term</th>
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</tr>
</thead>
<tbody>
<tr>
<td><strong>Property of Betweenness</strong> - Point</td>
<td></td>
</tr>
<tr>
<td>$M$ is between points $P$ and $Q$ if</td>
<td></td>
</tr>
<tr>
<td>and only if $PM + MQ = PQ$.</td>
<td></td>
</tr>
<tr>
<td><strong>Midpoint</strong> - The point between the</td>
<td></td>
</tr>
<tr>
<td>endpoints of a line segment that</td>
<td></td>
</tr>
<tr>
<td>divides the segment into two</td>
<td></td>
</tr>
<tr>
<td>equal halves.</td>
<td></td>
</tr>
<tr>
<td><strong>Segment Bisector</strong> - Any segment,</td>
<td></td>
</tr>
<tr>
<td>line or plane that intersects a</td>
<td></td>
</tr>
<tr>
<td>segment at its midpoint.</td>
<td></td>
</tr>
</tbody>
</table>

**Examples**

1. Find the value of $a$ and $XY$ if $Y$ is between $X$ and $Z$, $XY = 3a$, $XZ = 5a - 4$, and $YZ = 14$.

   $\begin{align*}
   X & \quad Y \quad Z \\
   3a & \quad 14 & 5a-4 \\
   \end{align*}$

   
   
   $\begin{align*}
   XY + YZ &= XZ \\
   3a + 14 &= 5a - 4 \\
   18 &= 2a \\
   a &= 9 \\
   XY &= 27
   \end{align*}$

2. Find $x$ and $BC$ if $B$ is between $A$ and $C$, $AC = 4x - 12$, $AB = x$, and $BC = 2x + 3$.

   $\begin{align*}
   A & \quad B \quad C \\
   x & \quad 2x+3 & 4x-12 \\
   \end{align*}$

   
   
   $\begin{align*}
   AB + BC &= AC \\
   x + 2x + 3 &= 4x - 12 \\
   3x + 3 &= 4x - 12 \\
   15 &= x \\
   BC &= 3x + 3 = 2(15) + 3 \\
   BC &= 33
   \end{align*}$

3. Line $k$ intersects line segment $\overline{AB}$ at point $D$. Point $D$ is between points $A$ and $B$.

   If $AD = 2$, $BD = x^2$ and $AB = x + 14$, find $x$.

   $\begin{align*}
   AD + DB &= AB \\
   2 + x^2 &= x + 14 \\
   0 &= x^2 - x - 12 \\
   0 &= (x - 4)(x + 3) \\
   x &= 4 \quad x = -3
   \end{align*}$
4. Line \( k \) bisects line segment \( \overline{AB} \). If \( AM = x - 7 \), and \( AB = 3x - 35 \), find the length of \( MB \).

\[
AM = MB, \quad AM = \frac{1}{2}(AB)
\]

\[
x - 7 = \frac{1}{2}(3x - 35)
\]

\[
2(x - 7) = 3x - 35
\]

\[
2x - 14 = 3x - 35
\]

\[
x = 21
\]

\[
MB = x - 7 = 21 - 7 = 14
\]

**Perpendicular Lines** - Lines, line segments or rays that intersect to form right angles.

**Perpendicular Bisector** - A line that is perpendicular to a line segment at its midpoint.

**Perpendicular Bisector Theorem** - A point is on the perpendicular bisector of a line segment if and only if it is equidistant from the endpoints of the line segment.

5. Line \( \overline{PQ} \) is the perpendicular bisector of line segment \( \overline{AB} \).
   a. Find \( x \):

\[
AM = MB
\]

\[
3x - 17 = 5x - 25
\]

\[
x = 4
\]

b. If \( m\angle PMA = 10y - 20 \), find \( y \).

\[
m\angle PMA = 90 \quad \text{2 lines intersect at point P forming rt. \& s's}
\]

\[
90 = 10y - 20
\]

\[
110 = 10y
\]

\[
y = 11
\]

4. Line \( \overline{PQ} \) is the perpendicular bisector of \( \overline{QT} \). Find \( QR \).

\[
QR = RT
\]

\[
2x + 3 = 4x - 7
\]

Since \( R \) lies on the \( \perp \) bisector of \( \overline{QT} \), it is equidistant from \( Q \) and \( T \).
## ANGLES

<table>
<thead>
<tr>
<th>Term</th>
<th>Diagram/Symbols</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Angle</strong> - The union of two non-collinear rays that share a common endpoint.</td>
<td>![Diagram of Angle]</td>
</tr>
<tr>
<td><strong>Interior</strong> - The interior of $\angle AOC$ is always the &quot;smaller&quot; region of the two regions defined by the angle (the region that is convex). The other region is called the exterior of the angle.</td>
<td>![Diagram of Interior]</td>
</tr>
<tr>
<td><strong>Vertex</strong> - The common endpoint shared by the rays (sides) of an angle.</td>
<td>![Diagram of Vertex]</td>
</tr>
<tr>
<td><strong>Degrees</strong> - Angle measure formed by dividing the distance around a circle into 360 parts.</td>
<td></td>
</tr>
<tr>
<td><strong>Opposite Rays</strong> - Two rays, sharing a common endpoint, with a 180 degree angle.</td>
<td>![Diagram of Opposite Rays]</td>
</tr>
</tbody>
</table>

### Examples

1. For the figure shown:
   a. Name two rays. $\overrightarrow{EF}$, $\overrightarrow{ED}$
   b. Name the vertex of the angle. Point $E$
   c. Name the sides of the angle. $\overrightarrow{EF}$, $\overrightarrow{ED}$
   d. Name the angle in four ways. $\angle FED$, $\angle DEF$, $\angle y$, $\angle E$

<table>
<thead>
<tr>
<th>Right Angle - An angle that measures 90 degree</th>
<th>![Right Angle Diagram]</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Acute Angle</strong> - An angle with measure less than 90 degrees.</td>
<td>![Acute Angle Diagram]</td>
</tr>
<tr>
<td><strong>Obtuse Angle</strong> - An angle with measure between 90 and 180 degrees.</td>
<td>![Obtuse Angle Diagram]</td>
</tr>
</tbody>
</table>
2. $D$ is a point in the interior of $\angle ABC$, $m\angle ABD = 15$, and $m\angle DBC = 90$.

   a. Find $m\angle ABC$.
   \[ m\angle ABC = m\angle ABD + m\angle DBC = 15 + 90 = 105 \]  
   b. Name an acute angle.
   \[ \angle ABD, \angle DBA \]
   c. Name a right angle.
   \[ \angle DBC, \angle CBD \]
   d. Name an obtuse angle.
   \[ \angle ABC, \angle CBA \]

**Angle Bisector** - A ray that divides an angle into two equal angles, each half the measure of the original angle.

3. In the figure, $\overline{CD}$ bisects $\angle ACB$.
   a. Name $\angle 1$ in two other ways.
   \[ \angle ACD, \angle BCA \]
   b. Write a conclusion that states that two angles are congruent.
   \[ \angle ACD \cong \angle BCD \]
   c. Write a conclusion that states that two angles have equal measures.
   \[ m\angle ACD = m\angle BCD \]

4. $\overline{PB}$ bisects $\angle APC$. If $m\angle APC = (x^2 - 2x)^\circ$ and $m\angle APB = (4x - 12)^\circ$, find $m\angle CPB$.

   \[
   m\angle APB = \frac{1}{2}(m\angle APC) \]
   
   \[
   4x - 12 = \frac{1}{2}(x^2 - 2x) \]
   
   \[
   2(4x - 12) = x^2 - 2x \]
   
   \[
   8x - 24 = x^2 - 2x \]
   
   \[
   8x - 24 = x^2 - 2x \]
   
   \[
   0 = x^2 - 10x + 24 \]
   
   \[
   0 = (x - 6)(x - 4) \]
   
   \[
   x = 6, x = 4 \]

   \[
   m\angle CPB = \frac{1}{2}(x - 12) \]
   
   \[
   m\angle CPB = \frac{4}{4}(4 - 12) \]
   
   \[
   = 4(6) - 12 \]
   
   \[
   = 16 - 12 \]
   
   \[
   = 4 \]
Copy the angle below:

Steps:

1. Label the vertex of the angle A

2. Draw a ray $\overrightarrow{EA}$ as one side of the angle to be drawn.

3. Draw circle $C_A$: center $A$, any radius.

4. Label the intersections of $C_A$ with the sides of the angle as $B$ and $C$.

5. Draw circle $C_E$: center $E$, radius $AB$.

6. Label the intersection of $C_E$ with $\overrightarrow{EA}$ as $F$.

7. Draw circle $C_E$: center $E$, radius $BC$.


9. Label either intersection of $C_E$ and $C_F$ as $D$.

10. Draw ray $\overrightarrow{ED}$.

11. 

12. 

13. 

14. 
Bisect the angle:

Steps:

1. Label the vertex of the angle A

2. *draw circle C_A: center A, any size radius*

3. *label intersections of C_A with rays of \( \angle BAC \) as B and C*

4. *draw circle C_B: center B, radius BC*

5. *draw circle C_C: center C, radius BC*

6. *label an intersection of C_B and C_C in the interior of \( \angle BAC \) as D*

7. *draw ray \( \overrightarrow{AD} \)*

8.

9.

10.

11.

12.

13.

14.

15.
TRIANGLES

**Polygon** - A closed figure formed by a finite number of coplanar segments.

**Vertex of a Polygon** - The intersection of two sides of a polygon.

Triangles can be classified in two ways – by their angles or by their sides. All triangles have at least two acute angles, but the third angle is used to classify the triangle.

<table>
<thead>
<tr>
<th>Classifications of Triangles By Angles</th>
<th>Figure</th>
<th>Characteristics</th>
</tr>
</thead>
<tbody>
<tr>
<td>Acute Triangle</td>
<td><img src="image" alt="Acute Triangle" /></td>
<td>A ( \triangle ) with 3 acute ( \angle )'s</td>
</tr>
<tr>
<td>Equiangular Triangle</td>
<td><img src="image" alt="Equiangular Triangle" /></td>
<td>A ( \triangle ) with 3 angles equal in measure ( m \times N = m \times M = m \times L = 60^\circ )</td>
</tr>
<tr>
<td>Obtuse Triangle</td>
<td><img src="image" alt="Obtuse Triangle" /></td>
<td>A ( \triangle ) with one obtuse ( \angle ) (and 2 acute ( \angle )'s)</td>
</tr>
<tr>
<td>Right Triangle</td>
<td><img src="image" alt="Right Triangle" /></td>
<td>A ( \triangle ) with one right ( \angle ) (and 2 acute ( \angle )'s)</td>
</tr>
</tbody>
</table>

**Examples**

Classify each triangle as acute, equiangular, obtuse, or right.

1. ![Triangle 1](image) \( \text{sinc} \ 97^\circ \neq 90^\circ \)
   - obtuse

2. ![Triangle 2](image) \( \text{equiangular} \)
   - sinc all \( \angle \)'s are \( = \) in measure

3. \( \triangle \)PQR
   - sinc \( m \times \angle \)PQR \( > 90^\circ \)
Triangles can also be classified according to the number of congruent sides they have.

<table>
<thead>
<tr>
<th>Classification of Triangles By Sides</th>
<th>Figure</th>
<th>Characteristics</th>
</tr>
</thead>
<tbody>
<tr>
<td>Equilateral Triangle</td>
<td><img src="image1" alt="Equilateral Triangle" /></td>
<td>A Δ with 3 equal sides</td>
</tr>
<tr>
<td>Isosceles Triangle</td>
<td><img src="image2" alt="Isosceles Triangle" /></td>
<td>A Δ with 2 equal sides</td>
</tr>
<tr>
<td>Scalene Triangle</td>
<td><img src="image3" alt="Scalene Triangle" /></td>
<td>A Δ with no equal sides</td>
</tr>
</tbody>
</table>

Examples
Classify each triangle as equilateral, isosceles, or scalene.

4. Isosceles Δ since 2 sides are equal in length

5. ΔJKM, if point M is the midpoint of JL

6. Find the measures of the sides of isosceles triangle ABC.

   \[ AC = BC \]

   \[ 4x + 1 = 5x - 0.5 \]

   \[ x = 1.5 \]

   \[ AB = 9(1.5) - 1 \]

   \[ AB = 12.5 \]

   \[ AC = 4(1.5) + 1 \]

   \[ AC = 7 \]

   \[ BC = 5(1.5) - 0.5 \]

   \[ BC = 7 \]

7. Find the measures of the sides of equilateral triangle FGH.

   \[ F6 = 6H = FH \]

   \[ 2y + 5 = 3y - 3 \]

   \[ 8 = y \]

   \[ F6 = 2(8) + 5 \]

   \[ F6 = 21 \]

   \[ FH = 21 \]

   \[ GH = 21 \]
8. In $\triangle DEF$, $m\angle E = 90$, and $EF = ED$.

   a. Classify the triangle according to sides.

   \[ \text{isosceles } \triangle \quad (2 = \text{sides}) \]

   b. Classify the triangle according to angles.

   \[ \text{right } \triangle \quad (\text{right } \ast) \]

   c. What sides are the legs of the triangle?

   $\overline{DE}, \overline{EF}$

   d. What side is opposite the right angle?

   $\overline{DF}$

   e. What angle is opposite $\overline{EF}$?

   $\angle D, \angle EDF, \angle FDE$

   f. What angle is included between $\overline{EF}$ and $\overline{FD}$?

   $\angle F, \angle EFD, \angle DFE$

Follow the steps for the construction of an equilateral triangle:

1. Draw circle $C_f$; center $J$, radius $JS$.
2. Draw circle $C_s$; center $S$, radius $SJ$.
3. Label the intersection as $M$.

![Equilateral Triangle Construction](image-url)
Cedar City boasts two city parks and is in the process of designing a third. The planning committee would like all three parks to be equidistant from one another to better serve the community. A sketch of the city appears below, with the centers of the existing parks labeled as $P_1$ and $P_2$. Identify two possible locations for the third park and label them as $P_{3a}$ and $P_{3b}$ on the map. Clearly and precisely list the mathematical steps used to determine each of the two potential locations.

Steps:

1. Draw a circle $C_{P1}$: center $P_1$, radius $P_1P_2$.

2. Draw a circle $C_{P2}$: center $P_2$, radius $P_2P_1$.

3. Label the two intersections of the circles $P_{3a}$, $P_{3b}$.
Using the skills you have practiced, **construct** three equilateral triangles, where the first and second triangles share a common side, and the second and third triangles share a common side. Clearly and precisely list the steps needed to accomplish this construction.

**Construction:**
**Steps:**

1. Draw a segment $AB$.
2. Draw circle $C_A$: center $A$, radius $AB$.
3. Draw circle $C_B$: center $B$, radius $BA$.
4. Label the intersections $C$, and $D$.
5. Draw circle $C_C$: center $C$, radius $CA$.
6. Label the new intersection of $C_C$ and $C_A$ as $E$.
THE ISOSCELES TRIANGLE THEOREM AND ITS CONVERSE [5]

<table>
<thead>
<tr>
<th>Isosceles Triangles</th>
<th>Words</th>
<th>Example</th>
<th>Figure</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Isosceles Triangle Theorem</strong></td>
<td>If two sides of a triangle are congruent, then their opposite angles are congruent.</td>
<td>If $\overline{AC} \cong \overline{BC}$ then $\angle 1 \cong \angle 2$</td>
<td><img src="" alt="Diagram" /></td>
</tr>
<tr>
<td><strong>Converse of the Isosceles Triangle Theorem</strong></td>
<td>If two angles of a triangle are congruent, then their opposite sides are congruent.</td>
<td>If $\angle 1 \cong \angle 2$, then $\overline{DE} \cong \overline{FE}$</td>
<td><img src="" alt="Diagram" /></td>
</tr>
</tbody>
</table>

Examples

a. Name the two unmarked congruent angles.
   $\angle B$ and $\angle ACB$

b. Name two unmarked congruent segments
   $\overline{AC}$ and $\overline{AD}$

1. Classify the triangle shown by its angles. Explain your reasoning.
   
   $15x + 1 + 6x + 5 + 4x - 1 = 180$
   
   $25x + 5 = 180$
   
   $25x = 175$
   
   $x = 7$

   The $\triangle$ is an obtuse $\triangle$ since it has one obtuse $\angle$ (an $\angle$ measuring $> 90^\circ$).

2. Find the value of each variable.

   a. $6x + 8 + 6x + 8 + 80 = 180$
      
      $12x + 16 + 80 = 180$
      
      $12x + 96 = 180$
      
      $12x = 84$
      
      $x = 7$

   The sum of the $\angle$s of a $\triangle$ is 180°. If 2 $\angle$s of a $\triangle$ are $\cong$ then their opp. $\angle$s are $\cong$. 
b. \[ \begin{align*}
2z - 15 &= 9 \\
2z &= 24 \\
z &= 12
\end{align*} \]

if 2 sides of a \( \Delta \) are \( \cong \), then their opp. sides are \( \cong \)

\[ 9 = 2z - 15 \]

\[ 2z = 24 \]

\[ z = 12 \]

c. \[ \begin{align*}
6x - 2 &= 4x \\
6x &= 4x + 2 \\
x &= 16
\end{align*} \]

if 2 sides of a \( \Delta \) are \( \cong \), then their opp. sides are \( \cong \)

\[ 6x - 2 = 4x \]

\[ 6x = 4x + 2 \]

\[ x = 16 \]

d. \[ \begin{align*}
2x + 11 &= 6x - 9 \\
20 &= 4x \\
x &= 5
\end{align*} \]

if 2 sides of a \( \Delta \) are \( \cong \), then their opp. sides are \( \cong \)

\[ 2x + 11 = 6x - 9 \]

\[ 20 = 4x \]

\[ x = 5 \]

e. \[ \begin{align*}
2(3x + 6) + 90 &= 180 \\
6x + 12 &= 90 \\
6x &= 78 \\
x &= 13
\end{align*} \]

if 2 sides of a \( \Delta \) are \( \cong \), then their opp. sides are \( \cong \)

the sum of the int. \( \Delta \)s of a \( \Delta \) is 180

\[ 2(3x + 6) + 90 = 180 \]

\[ 6x + 12 = 90 \]

\[ 6x = 78 \]

\[ x = 13 \]

f. \[ \begin{align*}
4x + 20 &= 6y - 2 \\
4x - 6y &= -22 \\
4x + 20 + 6y - 2 + 52 &= 180 \\
4x + 6y + 70 &= 180
\end{align*} \]

if 2 sides of a \( \Delta \) are \( \cong \), then their opp. sides are \( \cong \)

the sum of the int. \( \Delta \)s of a \( \Delta \) is 180

\[ 4x + 20 = 6y - 2 \]

\[ 4x - 6y = -22 \]

\[ 4x + 6y + 70 = 180 \]
TRIANGLE CENTERS

Recall, the perpendicular bisector of a segment is a line, ray, or line segment that is perpendicular to the given segment at its midpoint. A point is on the perpendicular bisector of a line segment if and only if it is equidistant from the endpoints of the line segment.

We now investigate how to construct a perpendicular bisector of a line segment using a compass and a straightedge. Follow the diagram and steps below:

1. Label the endpoints of the segment \( A \) and \( B \).
2. Draw circle \( C_A \) center \( A \), radius \( AB \) and circle \( C_B \) center \( B \), radius \( BA \).
3. Label the points of intersection as \( C \) and \( D \).
4. Draw line \( CD \).

Now that you are familiar with the construction of a perpendicular bisector, we must make one last observation. Using your compass, examine the following pairs of segments:

\[ AC, BC \]
\[ AD, BD \]
\[ AE, BE \]

Any point on the 1 bisector of a line segment is equidistant from the endpoints of the line segment.
**Concurrent Lines** - Three or more lines that intersect at a common point.

**Point of Concurrency** - The point where concurrent lines intersect.

**Circumcenter of a Triangle** - The point where the three perpendicular bisectors of a triangle meet.

**Circumcenter Theorem** - The circumcenter of a triangle is equidistant from the vertices of the triangle.

Construct the circumcenter of triangle ABC:

Steps:

1. draw circle $C_A$: center $A$, radius $AC$
2. draw circle $C_C$: center $C$, radius $CA$
3. label pts of intersection as $D$ and $E$
The question that arises here is **WHY** are the three perpendicular bisectors concurrent? Will these bisectors be concurrent in all triangles? To answer these questions, we must recall that all points on the perpendicular bisector are equidistant from the endpoints of the segment. This allows the following reasoning:

1. **P** is equidistant from **A** and **B** since it lies on the **perpendicular bisector** of **AB**.
2. **P** is also **equidistant** from **B** and **C** since it lies on the perpendicular bisector of **BC**.
3. Therefore, **P** must also be equidistant from **A** and **C**.

Hence, **AP = BP = CP**, which suggests that **P** is the point of **concurrency** of all three perpendicular bisectors.
**Angle Bisector Theorem**: If a point is on an angle bisector, then it is equidistant from the sides of the angle.

**Incenter** - The point of concurrency of the **angle bisectors** of a triangle.

**Incenter Theorem** - The incenter of a triangle is equidistant from the sides of a triangle.

Construct the incenter of triangle ABC:

**Steps:**

1. **draw circle $C_A$: center $A$, any size radius**
2. **label intersections of $C_A$ with rays of $\angle A$ as $D$ and $E$**
3. **draw circle $C_D$: center $D$, radius $DE$**
4. **draw circle $C_E$: center $E$, radius $ED$**
5. **label intersection of $C_E$ and $C_D$ as $F$**
6. **draw ray $\overrightarrow{AF}$**
7. Repeat steps 1-6 to construct a bisector of \( \overline{AC} \) and name \( \overline{CQ} \)

8. Label intersection of \( \overline{CQ} \) and \( \overline{AP} \) as \( P \)

9.

10.

The angle bisectors are always concurrent at the incenter.

Any point on the angle bisector is equidistant from the rays forming the angle. Therefore, since point \( Q \) is on the angle bisector of \( \angle ABC \), it is equidistant from \( BA \) and \( BC \). Similarly, since point \( Q \) is on the angle bisector of \( \angle BCA \), it is equidistant from \( CB \) and \( CA \). Therefore, \( Q \) must also be equidistant from \( AB \) and \( AC \), since it lies on the angle bisector of \( \angle BAC \). So \( Q \) is a point of concurrency of all three angle bisectors.

Observe the constructions below. Point \( A \) is the circumcenter of \( \triangle JKL \).

(Notice that it can fall outside of the triangle.) Point \( B \) is the incenter of \( \triangle RST \).

The circumcenter of a triangle is the center of the circle that circumscribes that triangle. The incenter of the triangle is the center of the circle that is inscribed in that triangle.

How can you use what you have learned in this lesson to find the center of a circle if the center is not shown?

Inscribe a \( \triangle \) into the circle and construct the \( 1 \) bisectors of at least \( 2 \) sides. The intersection is the center of the circle.